MCF 3M Culminating Task Practice

1. For the function $f(x) = -2x^2 + 4x + 1$, determine and simplify fully:

a) f(6) $= -2(6)^2 + 4(6) + 1$

$= -2(36) + 24 + 1$

$= -72 + 24 + 1$

$= -47$

b) f(-2) $= -2(-2)^2 + 4(-2) + 1$

$= -2(4) - 8 + 1$

$= -8 - 8 + 1$

$= -15$

2. Solve the following, using the quadratic formula, rounding to 1 decimal place:

$9x^2 - 7x + 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(9)(1)}}{2(9)}$$

$$x = \frac{7 \pm \sqrt{13}}{18}$$

$$x = \frac{7 + \sqrt{13}}{18}$$

$x = 0.8$

$$x = \frac{7 - \sqrt{13}}{18}$$

$x = 0.2$

3. Factor and solve for x:

a) $x^2 - 9x + 20$

$= (x - 5)(x - 4)$

$x = 5 \qquad x = 4$

b) $x^2 + 4x - 21$

$= (x + 7)(x - 3)$

$x = -7 \qquad x = 3$

4. The arch of the Tyne bridge in England is modelled by $h = -0.008x^2 - 1.29x + 107.5$, where h is the height of the arch above the riverbank and x is the horizontal distance from the riverbank, both in metres. Determine the maximum height of the arch.

max height $x = -\frac{b}{2a}$

$h = -0.008(-80.625)^2 - 1.29(-80.625) + 107.5$

$x = \frac{1.29}{2(-0.008)}$

$h = 159.5\text{ m}$

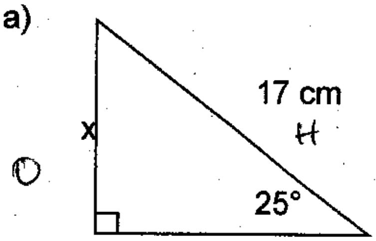
max height

$x = \frac{1.29}{-0.016}$

$x = -80.625$

5. Solve for the missing side length. Round to 1 decimal place

a)

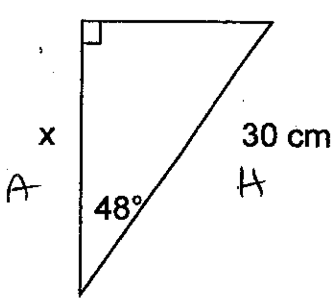


$\sin 25 = \frac{x}{17}$

$x = 17 \sin 25$

$x = 7.2\text{ cm}$

b)

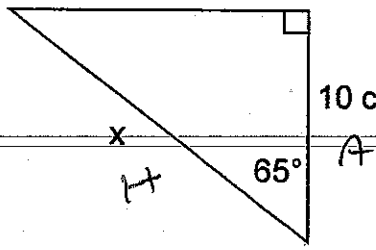


$\cos 48 = \frac{x}{30}$

$x = 30 \cos 48$

$x = 20.1\text{ cm}$

c)

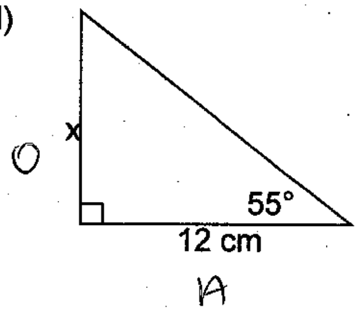


$\cos 65 = \frac{10}{x}$

$x = \frac{10}{\cos 65}$

$x = \cancel{17.9\text{ cm}}$ 23.7 cm

d)

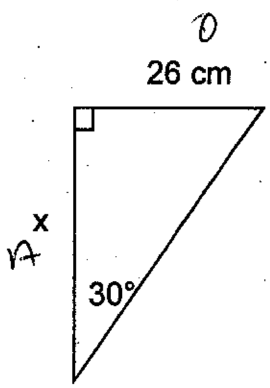


$\tan 55 = \frac{x}{12}$

$x = 12 \tan 55$

$x = 17.1$ cm

e)

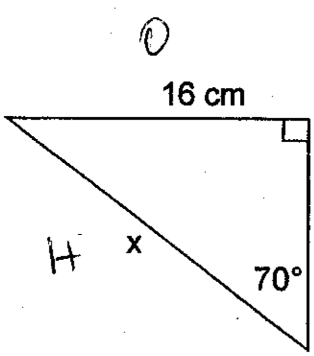


$\tan 30 = \frac{26}{x}$

$x = \frac{26}{\tan 30}$

$x = 45$ cm

f)



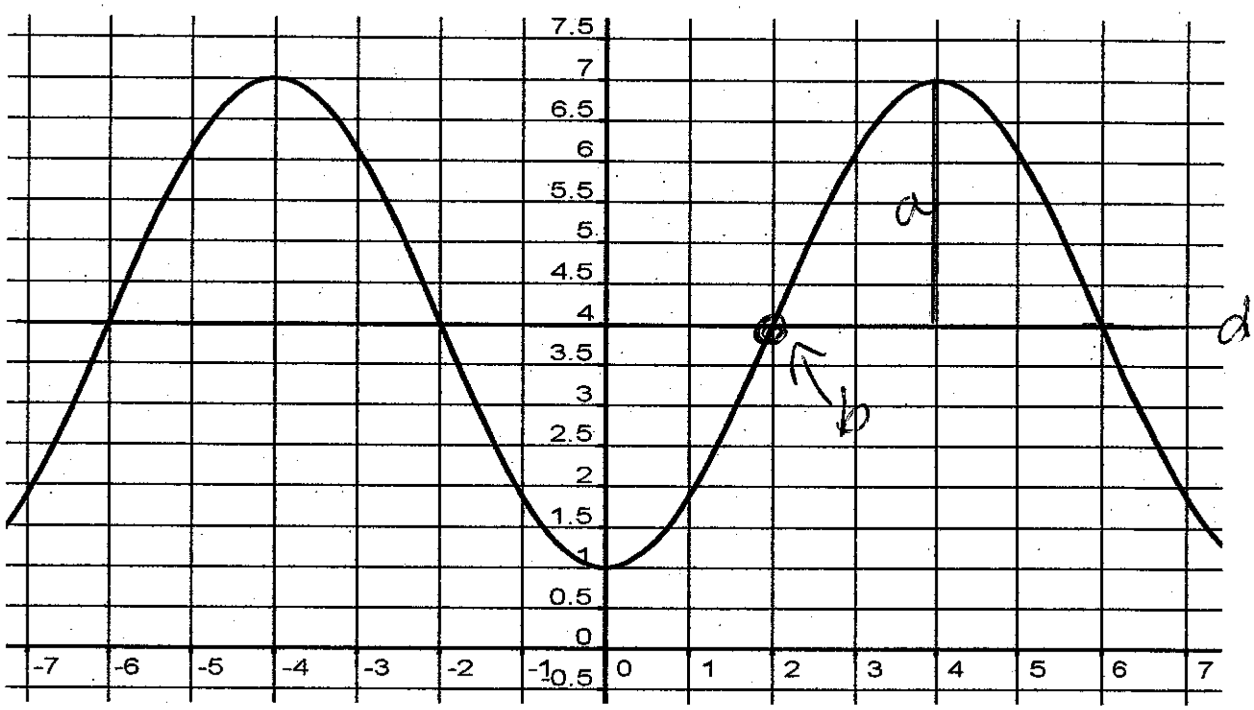
$\sin 70 = \frac{16}{x}$

$x = \frac{16}{\sin 70}$

$x = 17$ cm

6. Determine the parameters a, k, b, and d, and write the equation for the following curve. Test at least one point to show that your equation works.

$$y = a\sin(k(x-b)) + d$$



max = 7

min = 1

$a = \frac{7-1}{2}$

$a = 3$

$d = \frac{7+1}{2}$

$d = 4$

$k = \frac{360}{8}$

$k = 45$

$b = 2$

$y = 3 \sin 45(x-2) + 4$

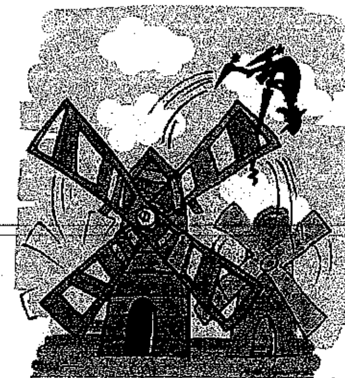
test (0, +1)

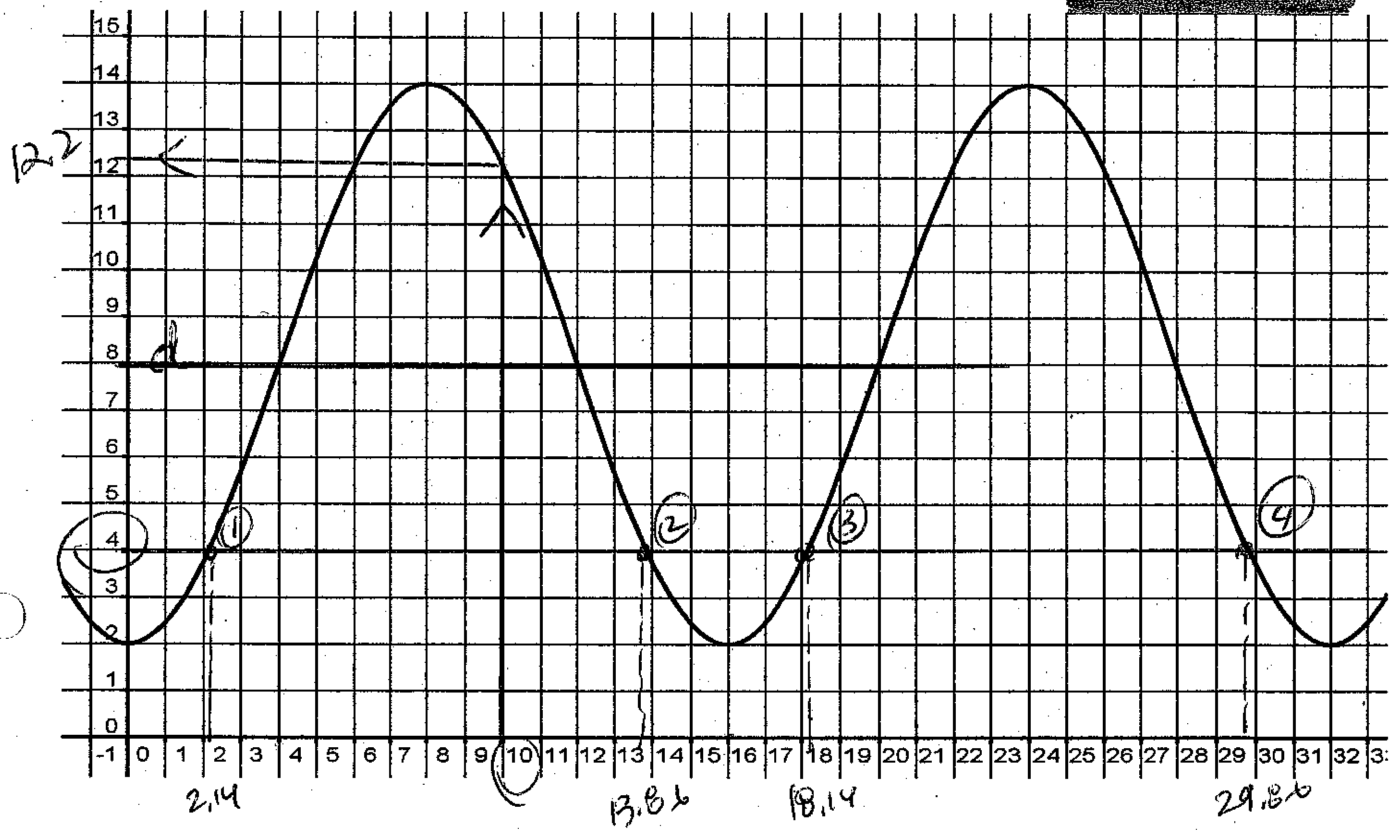
$y = 3\sin 45(0-2) + 4$

$y = 3\sin(-90) + 4$

$y = 1$ ✓

7. Don Quixote, a fictional character in a Spanish novel, attacked windmills, thinking they were giants. At one point, he got snagged by one of the blades and was hoisted into the air. The graph shows his height above the ground in terms of time.





a) Determine the equation of this curve. Test a point to show that it works.

max = 14

min = 2

$a = \frac{14-2}{2}$

$a = 6$

$d = \frac{14+2}{2}$

$d = 8$

$k = \frac{360}{16}$

$k = 22.5$

$b = 4$

$y = 6 \sin 22.5(x-4) + 8$

test (0, 2)

$y = 6\sin 22.5(0-4) + 8$

$y = 6\sin(-90) + 8$

$y = 2$ ✓

b) Using your equation, determine Don's height above the ground after 10 seconds. Verify on the graph.

$y = 6\sin 22.5(10-4)+8$

$y = 6\sin 22.5(6)+8$

$y = 6\sin 135 + 8$

$y = 12.2$ m

verifies on graph ✓

c) During the first 32 seconds, how many times is Don exactly 4 metres above the ground?

4 times

d) Use your equation and your understanding of the symmetry in a periodic graph to determine when Don is 4 m above the ground. Verify with the graph.

$4 = 6\sin 22.5(x-4)+8$

$4-8 = 6\sin 22.5(x-4)$

$\frac{-4}{6} = \frac{6\sin 22.5(x-4)}{6}$

$-0.67 = \sin 22.5(x-4)$

$\frac{\sin^{-1}(-0.67)}{22.5} + 4 = x$

$2.14 = x$

all solutions are 2.14 to the left and right of every min

① 2.14 seconds

② min at 16
$16-2.14 = 13.86$ s

③ $16+2.14 = 18.14$ s

④ min at 32
$32-2.14 = 29.86$ s

pts verify on graph ✓

8. The population of a small town has increased at a rate of 1.5% per year since 1980. The population of the town in 1980 was 1600.

a) Write an equation that models the population of the town after n years.

$y = 1600(1.015)^n$ ← # years since 1980

popn. in 1980

growth

b) Use your equation to determine the population of the town in 2008.

$n = 2008 - 1980$

$n = 28$

$y = 1600(1.015)^{28}$

$y = 2427$ people

9. The population of a small mining town was 13 700 in 2000. Each year, the population decreases by 5%.

In what year will the population reach 5000 people?

$$\frac{5000}{13700} = \frac{13700(0.95)^n}{13700}$$

$$0.36496 = (0.95)^n$$

$$\frac{\log 0.36496}{\log 0.95} = n$$

$$19.65 = n$$

$2000 + 19.65$

$= 2019.65$

∴ part way through 2019

10. If $500 grows to $3000 in 5 years, what is the annual interest rate, assuming that interest is compounded monthly?

$$3000 = 500\left(1+\frac{i}{12}\right)^{12\times 5}$$

$$\frac{3000}{500} = \frac{500\left(1+\frac{i}{12}\right)}{500}$$

$$6 = \left(1+\frac{i}{12}\right)^{60}$$

$$\left[\left(6^{1/60}\right)-1\right] \times 12 = i$$

$$0.364 = i$$

$$36.4\%/a$$

11. If $300 grows into $1200 in 5 years, what is the annual interest rate, assuming interest is compounded quarterly?

$$1200 = 300\left(1+\frac{i}{4}\right)^{4\times 5}$$

$$\frac{1200}{300} = \frac{300\left(1+\frac{i}{4}\right)}{300}$$

$$4 = \left(1+\frac{i}{4}\right)^{20}$$

$$\left[\left(4^{1/20}\right)-1\right] \times 4 = i$$

$$0.287 = i$$

$$28.7\%/a$$

12. Determine the amount of the annuity with regular deposits of $500 every 6 months for 4 years at 8%/a compounded semi-annually.

$R = 500$

$i = \frac{0.08}{2}$

$i = 0.04$

$n = 4 \times 2$

$n = 8$

$FV = 500 \times \frac{(1.04)^8 - 1}{0.04}$

$FV = \$4607.11$

Investment: 500×8
$= \$4000$

Interest: $607.11

13. Determine the amount of the annuity with regular deposits of $200 every month for 8 years at 10%/a compounded monthly.

$R = 200$

$i = \frac{0.1}{12}$

$i = 0.0083$

$n = 8 \times 12$

$n = 96$

$FV = 200 \times \frac{(1.0083)^{96} - 1}{0.0083}$

$FV = \$29\,236.22$

Investment: 200×96
$= \$19\,200$

Interest $10 036.22