MCR 3U Final Exam Practice

1. Quadratics

Fully factor each of the following:

a) $x^2 - 6x - 27$

$= (x-9)(x+3)$

b) $4x^2 + 20x + 25$

$= (2x+5)^2$

c) $25x^2 - 49$

$= (5x-7)(5x+7)$

d) $6x^2 - x - 2$

$= (3x-2)(2x+1)$

e) $2x^2 - 7x - 15$

$= (2x+3)(x-5)$

f) $2x^2 - 2x - 24$

$= 2(x-4)(x+3)$

g) $49x^2 - 42x + 9$

$= (7x-3)^2$

h) $4x^2 - 49$

$= (2x-7)(2x+7)$

i) $2x^2 + 3x + 1$

$= (2x+1)(x+1)$

j) $8x^2 - 2x - 21$

$= (4x-7)(2x+3)$

k) $10x^2 - 28x + 16$

$= 2(5x-4)(x-2)$

l) $7x^2 - 28$

$= 7(x-2)(x+2)$

m) $x^2 + 2x - 24$

$= (x+6)(x-4)$

n) $5x^2 - 20x$

$= 5x(x-4)$

o) $-2x^2 + 8x + 42$

$= -2(x-7)(x+3)$

The monthly sales, n, in thousands of units, of a new perfume can be modelled by the function $n(t) = -0.75t^2 + 6t + 51$, where $t = 0$ corresponds to the first month of sales. Can the company expect to sell 61 000 units in any one month over the next year? Explain your answer clearly.

$t = \frac{-6}{2(-0.75)}$

$t = 4$

$n(4) = -0.75(4)^2 + 6(4) + 51$

$= 63$

∴ max sales are 63 000 units in the 4th month

∴ yes they can sell 61 000 units.

OR

$61 = -0.75t^2 + 6t + 51$

$0 = -0.75t^2 + 6t - 10$

$b^2 - 4ac = (6)^2 - 4(-0.75)(-10)$

$= 6$

positive ∴ yes it is possible.

If a rock were thrown off a spacecraft on Mars, its height, h, in metres above the ground can be modelled by $h = -1.9t^2 + 17.86t + 3.03$, where t is time in seconds. When would the rock reach a height of 2.1 m? (round to 2 decimal places)

$2.1 = -1.9t^2 + 17.86t + 3.03$

$0 = -1.9t^2 + 17.86t + 0.93$

$t = \frac{-17.86 \pm \sqrt{(17.86)^2 - 4(-1.9)(0.93)}}{2(-1.9)}$

$t = \frac{-17.86 \pm \sqrt{326.0476}}{-3.8}$

$\cancel{t = -0.05\,s}$ $t = 9.45\,s$

∴ it would reach a height of 2.1 m at $t = 9.45$ seconds

2. Inequalities and Radicals

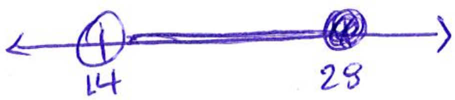
Solve each of the following inequalities. Express your solutions using a number line.

a) $1 < \frac{2x}{7} - 3 \leq 5$

$4 < \frac{2x}{7} \leq 8$

$28 < 2x \leq 56$

$14 < x \leq 28$



b) $-3 \leq \frac{4-x}{5} \leq 2$

$-15 \leq 4-x \leq 10$

$-19 \leq -x \leq 6$

$19 \geq x \geq -6$

or $-6 \leq x \leq 19$

Simplify each of the following radical expressions:

a) $4\sqrt{72} - 3\sqrt{27} + 2\sqrt{48} - \sqrt{18}$

$= 4\sqrt{36 \times 2} - 3\sqrt{9 \times 3} + 2\sqrt{16 \times 3} - \sqrt{9 \times 2}$

$= 24\sqrt{2} - 9\sqrt{3} + 8\sqrt{3} - 3\sqrt{2}$

$= 21\sqrt{2} - \sqrt{3}$

b) $-3\sqrt{200} + 4\sqrt{98} - 2\sqrt{50}$

$= -3\sqrt{100 \times 2} + 4\sqrt{49 \times 2} - 2\sqrt{25 \times 2}$

$= -30\sqrt{2} + 28\sqrt{2} - 10\sqrt{2}$

$= -12\sqrt{2}$

3. Functions and Function Notation

For g(x) = 3 - 2x, find

a) $g(3) = 3 - 2(3)$

$= 3 - 6$

$= -3$

b) $2g(1) = 2(3 - 2(1))$

$= 2(3-2)$

$= 2(1)$

$= 2$

c) $g(-2) - 3$

$= 3 - 2(-2) - 3$

$= 3 + 4 - 3$

$= 4$

For $f(x) = 2x^2 - 6x + 1$, find

a) $f(-2) = 2(-2)^2 - 6(-2) + 1$

$= 8 + 12 + 1$

$= 21$

b) $f(x+1) = 2(x+1)^2 - 6(x+1) + 1$

$= 2(x^2+2x+1) - 6x - 6 + 1$

$= 2x^2 + 4x + 2 - 6x - 5$

$= 2x^2 - 2x - 3$

List the transformations and then state Domain and Range:

a) $y = \frac{1}{8}\sqrt{-3x+12} + 5$

$-3(x-4)$

- v comp 1/8
- refl in y axis
- h comp 1/3
- right 4
- up 5

D: $\{x \mid x \leq 4, x \in R\}$

R: $\{y \mid y \geq 5, y \in R\}$

b) $y = \frac{7}{3}|-5(x+3)| - 4$

- v stretch 7/3
- refl in y axis
- h comp 1/2
- left 3
- down 4

D: $\{x \mid x \in R\}$

R: $\{y \mid y \geq -4, y \in R\}$

c) $y = \frac{1}{-2(x+4)} + 1$

- refl in y axis
- h comp 1/2
- left 4
- up 1

D: $\{x \mid x \neq -4, x \in R\}$

R: $\{y \mid y \neq 1, y \in R\}$

d) $y = -5\sqrt{-0.5x-5} - 3$

$-0.5(x+10)$

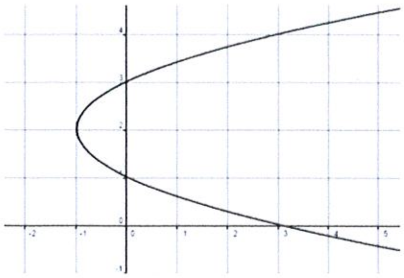
- refl in x axis
- v stretch 5
- refl in y axis
- h stretch 2
- left 10
- down 3

D: $\{x \mid x \leq -10, x \in R\}$

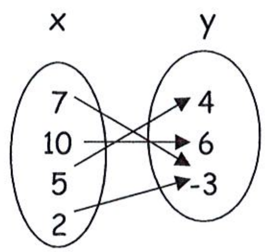
R: $\{y \mid y \leq -3, y \in R\}$

State whether any of the following relations are functions by answering "yes" or "no" in the space below. Then provide a reason for your choice.

a)



b)



c) $\{(-3,6),(-1,1),(0,1),(-3,8)\}$

a) Function? NO

Reason: fails VLT

b) Function? yes

Reason: no duplication of x values

c) Function? No

Reason: The x value -3 has two different y values.

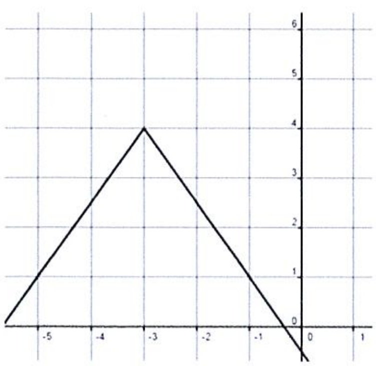
Fill in the chart (next page) that matches the equation to the graph. NOTE: There are extra equations.

1. $y=-\frac{1}{2}\sqrt{2(x-3)}+4$ **2.** $y=-\frac{2}{x+1}-3$ **3.** $y=-3\left|\frac{1}{2}(x-3)\right|+4$ **4.** $y=-2\sqrt{x+3}+4$

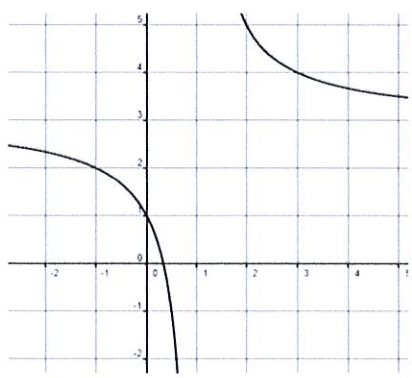
5. $y=\frac{2}{(x-1)}+3$ **6.** $y=3\left|\frac{1}{2}(x-3)\right|+4$ **7.** $y=-2\sqrt{-0.3(x-4)}+3$ **8.** $y=\frac{2}{-(x+1)}+3$

9. $y=3\left|\frac{1}{2}(x+3)\right|+4$ **10.** $y=\frac{1}{2}\sqrt{-(x-4)}-3$ **11.** $y=\frac{2}{-(x-1)}-3$ **12.** $y=-3\left|\frac{1}{2}(x+3)\right|+4$

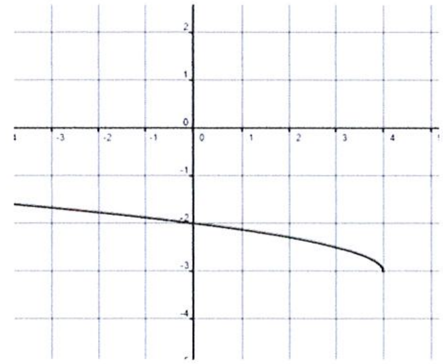
a)



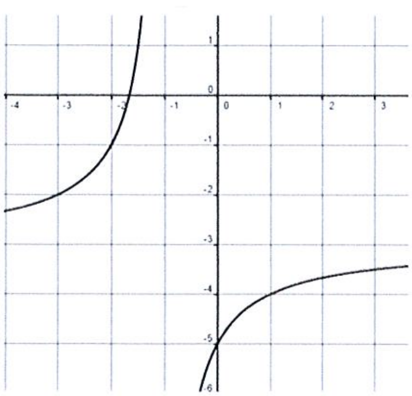
b)



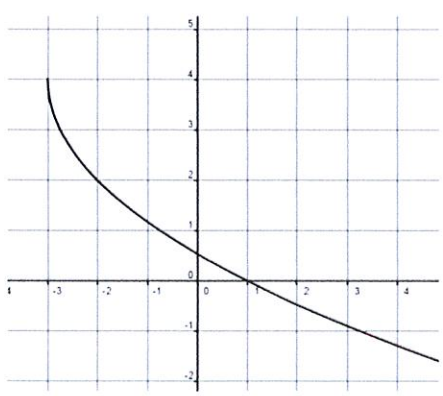
c)



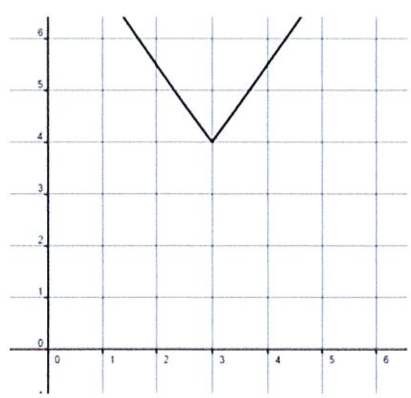
d)



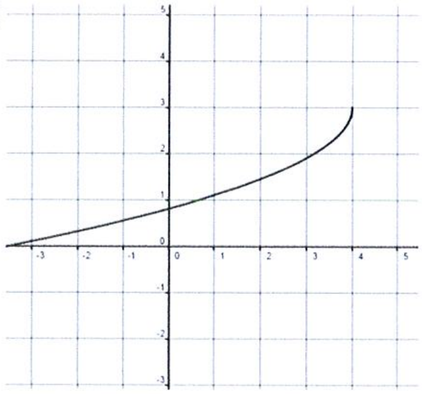
e)



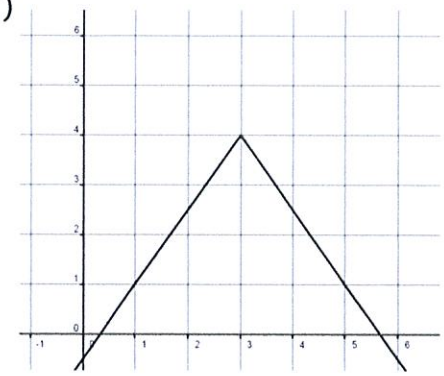
f)



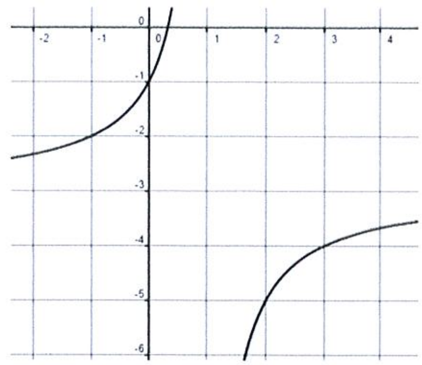
g)



h)



i)



Graph	a	b	c	d	e	f	g	h	i
Matches Equation	12	5	10	2	1	6	7	3	11

4. Inverse and Rational Functions

Given $f(x) = \frac{4}{5}x - 1$, evaluate:

inverse

$$y = \frac{4}{5}x - 1$$
$$x = \frac{4}{5}y - 1$$
$$x + 1 = \frac{4}{5}y$$
$$\frac{5(x+1)}{4} = y$$
$$\therefore f^{-1}(x) = \frac{5(x+1)}{4}$$

a) $f^{-1}(-9)$

$$= \frac{5(-9+1)}{4}$$
$$= \frac{5(-8)}{4}$$
$$= -10$$

b) $f(10)$ $= \frac{4}{5}(10) - 1$

$$= 8 - 1$$
$$= 7$$

Given $f(x) = \frac{4x-1}{5}$, evaluate

inverse

$$y = \frac{4x-1}{5}$$
$$x = \frac{4y-1}{5}$$
$$5x = 4y - 1$$
$$5x + 1 = 4y$$
$$\frac{5x+1}{4} = y$$
$$\therefore f^{-1}(x) = \frac{5x+1}{4}$$

a) $f(9)$

$$= \frac{4(9)-1}{5}$$
$$= \frac{35}{5}$$
$$= 7$$

b) $f^{-1}(3)$ $= \frac{5(3)+1}{4}$

$$= \frac{16}{4}$$
$$= 4$$

Determine the inverse of the function P(x) = -4.5(x - 8)² + 11

$$P-11=-4.5(x-8)^2$$

$$\frac{P-11}{-4.5}=(x-8)^2$$

$$8\pm\sqrt{\frac{P-11}{-4.5}}=x$$

Simplify, stating any restrictions on the variables

a) $\frac{3}{x^2-9}\div\frac{3x-6}{x-3}$

$$=\frac{\cancel{3}}{\cancel{(x-3)}(x+3)}\times\frac{\cancel{(x-3)}}{\cancel{3}(x-2)}$$

$$=\frac{1}{(x+3)(x-2)},\ x\neq\pm3,2$$

b) $\frac{3}{x^2-2x-15}\times\frac{x^2-9}{6x-18}$

$$=\frac{\cancel{3}^{1}}{(x-5)\cancel{(x+3)}}\times\frac{\cancel{(x-3)}\cancel{(x+3)}}{\cancel{6}_{2}\cancel{(x-3)}}$$

$$=\frac{1}{2(x-5)},\ x\neq\pm3,5$$

c) $\frac{2x^2-3x-2}{6x^2-x-2}\times\frac{3x^2-5x+2}{4x^2-1}\div\frac{x^2-1}{6x^2+7x+2}$

$$=\frac{\cancel{(2x+1)}(x-2)}{\cancel{(3x-2)}\cancel{(2x+1)}}\times\frac{\cancel{(3x-2)}\cancel{(x-1)}}{(2x-1)\cancel{(2x+1)}}\times\frac{(3x+2)\cancel{(2x+1)}}{\cancel{(x-1)}(x+1)}$$

$$=\frac{(x-2)(3x+2)}{(2x-1)(x+1)},\ x\neq\pm\frac{2}{3},\pm\frac{1}{2},\pm1$$

a) hole @ $x=3$
VA @ $x=-3$
$x=2$

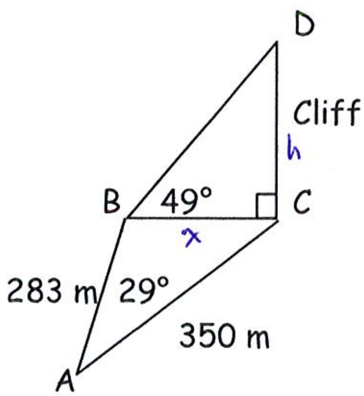
b) hole @ $x=3$
$x=-3$
VA @ $x=5$

c) hole @ $x=\frac{2}{3}$
$x=-\frac{1}{2}$
$x=1$
VA @ $x=-\frac{2}{3}$
$x=\frac{1}{2}$
$x=-1$

Choose one (or more) of the above and identify the restrictions as either holes or asymptotes.

5. Trig Problem Solving

An engineer needs to find the height of a cliff and takes the measurements shown. How high is the cliff?



$x^2 = 283^2 + 350^2 - 2(283)(350)\cos 29$

$x = 171.3\text{m}$

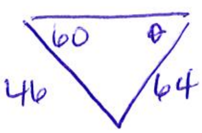
$\tan 49 = \dfrac{h}{171.3}$

$h = 171.3\tan 49$

$h = 197\text{ m}$ ← height of cliff.

A chandelier is suspended from a ceiling by two chains. One chain is 46 cm long and forms an angle of 60° with the ceiling. The other chain is 64 cm long. What angle does the longer chain make with the ceiling? (round to the nearest tenth)

SSA but solving for an angle ∴ not amb. case

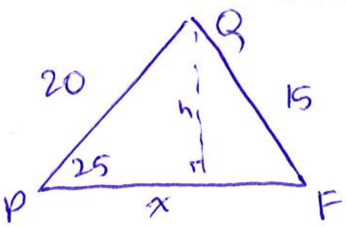


$\dfrac{\sin\theta}{46} = \dfrac{\sin 60}{64}$

$\theta = \dfrac{46\sin 60}{64}$

$\theta = 38.5°$

Two forest fire stations, P and Q, are 20 km apart. A ranger at station Q sees a fire 15 km away. If the angle between the line PQ and the line from P to the fire is 25°, how far, to the nearest tenth of a kilometer, is station P from the fire?



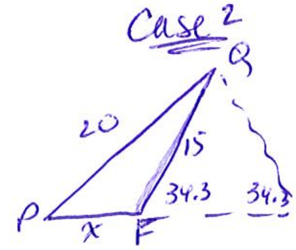
SSA ∴ check height

$h = 20\sin 25$

$h = 8.5$

$h < 15 < 20$ ∴ 2 Solutions

Case 2



$F = 180 - 34.3$

$F = 145.7°$

∴ $Q = 9.3°$

Case 1 $\dfrac{\sin F}{20} = \dfrac{\sin 25}{15}$

$F = \sin^{-1}\left(\dfrac{20\sin 25}{15}\right)$

$F = 34.3°$

∴ $Q = 120.7°$

$\dfrac{x}{\sin 120.7} = \dfrac{15}{\sin 25}$

$x = \dfrac{15\sin 120.7}{\sin 25}$

$x = 30.5\text{ km}$

$\dfrac{x}{\sin 9.3} = \dfrac{15}{\sin 25}$

$x = \dfrac{15\sin 9.3}{\sin 25}$

$x = 5.7\text{ km}$

∴ station P is either 30.5 km or 5.7 km from the fire.

6. Trigonometry

Find the simplified **exact** value of each of the following (express your answer as a single fraction, no decimals):

a) $\cos 45° + 2 \tan 60° \sin 60° - \cos 60°$

$= \frac{\sqrt{2}}{2} + 2(\sqrt{3})\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{2}$

$= \frac{\sqrt{2}}{2} + \frac{6}{2} - \frac{1}{2}$

$= \frac{\sqrt{2}+5}{2}$

b) $\sin^2 45° - \frac{\tan 30°}{\cos 30°} + \sin 30°$

$= \left(\frac{1}{\sqrt{2}}\right)^2 - \frac{1}{\sqrt{3}} \div \frac{\sqrt{3}}{2} + \frac{1}{2}$

$= \frac{1}{2} - \frac{1}{\sqrt{3}} \times \frac{2}{\sqrt{3}} + \frac{1}{2}$

$= 1 - \frac{2}{3}$

$= \frac{3}{3} - \frac{2}{3}$

$= \frac{1}{3}$

Solve for x to one tenth of a degree, where $0° \leq x \leq 360°$

$10 \sin x - 3 = 5$

$\frac{10 \sin x}{10} = \frac{8}{10}$

$\sin x = \frac{4}{5}$ positive

S | A
T | C

$x = \sin^{-1}\left|\frac{4}{5}\right|$

$x = 53.1°$

Quad I : $x_1 = 53.1°$

Quad II : $x_2 = 180 - 53.1$
$= 126.9°$

$4 \cos x + 1 = 0$

$\frac{4 \cos x}{4} = \frac{-1}{4}$

$\cos x = \frac{-1}{4}$ negative

S | A
T | C

$x = \cos^{-1}\left|-\frac{1}{4}\right|$

$x = 75.5°$

Quad II : $x_1 = 180 - 75.5$
$= 104.5°$

Quad III : $x_2 = 180 + 75.5$
$= 255.5°$

Prove the following identities:

a) $1+cos\theta = \frac{\sin^2\theta}{1-cos\theta}$

$LS = 1+\cos\theta$

$$RS = \frac{\sin^2\theta}{1-\cos\theta}$$
$$= \frac{1-\cos^2\theta}{1-\cos\theta}$$
$$= \frac{(1-\cancel{\cos\theta})(1+\cos\theta)}{(1-\cancel{\cos\theta})}$$
$$= 1+\cos\theta$$

$LS = RS$

b) $tan\theta = \frac{sin\theta+\sin^2\theta}{cos\theta+sin\theta cos\theta}$

$LS = \tan\theta$
$$= \frac{\sin\theta}{\cos\theta}$$

$$RS = \frac{\sin\theta(1+\cancel{\sin\theta})}{\cos\theta(1+\cancel{\sin\theta})}$$
$$= \frac{\sin\theta}{\cos\theta}$$

$LS = RS$

c) $\frac{sin\theta}{sin\theta+cos\theta} = \frac{tan\theta}{1+tan\theta}$

$$LS = \frac{\sin\theta}{\sin\theta+\cos\theta}$$

$$RS = \tan\theta \div 1+\tan\theta$$
$$= \frac{\sin\theta}{\cos\theta} \div 1+\frac{\sin\theta}{\cos\theta}$$
$$= \frac{\sin\theta}{\cos\theta} \div \frac{\cos\theta+\sin\theta}{\cos\theta}$$
$$= \frac{\sin\theta}{\cancel{\cos\theta}} \times \frac{\cancel{\cos\theta}}{\cos\theta+\sin\theta}$$
$$= \frac{\sin\theta}{\cos\theta+\sin\theta}$$

$LS = RS$

d) $\frac{\cos^2\theta+\cos\theta}{\sin^2\theta} = \frac{\cos\theta}{1-cos\theta}$

$$LS = \frac{\cos\theta(\cos\theta+1)}{\sin^2\theta}$$
$$RS = \frac{\cos\theta}{1-\cos\theta}$$

$$= \frac{\cos\theta(\cos\theta+1)}{1-\cos^2\theta}$$
$$= \frac{\cos\theta(\cancel{\cos\theta+1})}{(1-\cos\theta)(\cancel{1+\cos\theta})}$$
$$= \frac{\cos\theta}{1-\cos\theta}$$

$LS = RS$

e) $\frac{1-2\sin^2\theta}{sin\theta cos\theta} = \frac{1}{\tan\theta} - tan\theta$

$$LS = \frac{1-2\sin^2\theta}{\sin\theta\cos\theta}$$

$$RS = \cot\theta - \tan\theta$$
$$= \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$$
$$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{1-\sin^2\theta-\sin^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{1-2\sin^2\theta}{\sin\theta\cos\theta}$$

$LS = RS$

6. Periodic Functions

Complete the chart for the given sinusoidal functions. For the Phase Shift and Vertical Shift, give a direction and a number.

Function	Amplitude	Period (in degrees)	Max	Min	Phase Shift	Vertical Shift
$y = 4sin\left(\frac{1}{2}\theta - 30\right) + 1$	4	720°	5	-3	60° right	up 1
$y = -0.5cos(5\theta + 225) - 2.5$	$\frac{1}{2}$	72°	-2	-3	45° left	down 2.5
$y = -2sin(3(\theta - 30)) + 4$	2	120°	6	2	30° right	up 4

7. Exponential Functions

Simplify fully, expressing your answer with positive exponents and as a fraction, where appropriate.

a) $\dfrac{(64x^6b^3)^{\frac{1}{3}}}{(25x^{-4}b^6)^{\frac{1}{2}}}$

$= \dfrac{64^{1/3}x^2b^1}{25^{1/2}x^{-2}b^3}$

$= \dfrac{4x^4}{5b^2}$

b) $\left(\dfrac{(6x^3)(6y^3)^2}{(9xy)^6}\right)^{-\frac{1}{3}}$

$= \left(\dfrac{9^6x^6y^6}{(6x^3)(6^2y^6)}\right)^{1/3}$

$= \left(\dfrac{9^6x^6y^6}{6^3x^3y^6}\right)^{1/3}$

$= \left(\dfrac{9^6x^3}{6^3}\right)^{1/3}$

$= \dfrac{9^2x}{6}$

$= \dfrac{81x}{6}$

$= \dfrac{27x}{2}$

c) $\sqrt[4]{\dfrac{(5x^8)^3}{(5x^4)^{-1}}}$

$= \left(\dfrac{5^3x^{24}}{5^{-1}x^{-4}}\right)^{1/4}$

$= \left(5^4x^{28}\right)^{1/4}$

$= 5x^7$

Solve the following exponential equations:

a) $5^{2x-1} = \frac{1}{125}$

$5^{2x-1} = 5^{-3}$

$\therefore 2x-1 = -3$

$2x = -2$

$x = -1$

b) $3^{x^2} = 27(3^{2x})$

$3^{x^2} = 3^3(3^{2x})$

$3^{x^2} = 3^{3+2x}$

$\therefore x^2 = 3+2x$

$x^2-2x-3=0$

$(x-3)(x+1)=0$

$x=3 \quad x=-1$

8. Sequences and Series

For each of the following sequences, find t_n

-40, -25, -10, 5, ...

$a=-40$ $\quad t_n = -40+(n-1)(15)$

$d=15$ $\quad = -40+15n-15$

$t_n = -55+15n$

2, 14, 98, 686, ...

$a=2$

$r=7$ $\quad t_n = 2(7)^{n-1}$

Find S_{17} 32 + 43 + 54 + ...

$a=32$

$d=11$

$S_{17} = \frac{17}{2}(2(32)+(17-1)(11))$

$= 8.5(64+176)$

$\therefore S_{17} = 2040$

Find the sum of the geometric series 192 000 – 96 000 + 48 000 - ... + 1500

$a = 192\,000$

$r=-1/2$

$t_n = 1500$

$1500 = 192000\left(-\frac{1}{2}\right)^{n-1}$

$\frac{1500}{192000} = \left(-\frac{1}{2}\right)^{n-1}$

$n = \frac{\log\left(\frac{1500}{192000}\right)}{\log\left|-\frac{1}{2}\right|}+1$

$n = 8$ terms

$S_8 = \frac{192000\left(\left(-\frac{1}{2}\right)^8-1\right)}{-\frac{1}{2}-1}$

$\therefore S_8 = 127\,500$